

Tit-for-tat no SPNE?

Let $0 < \delta < 1, T \geq 0$:

$$\begin{aligned}
 1 &\geq (1-\delta) \cdot \left(\sum_{t=0}^{\infty} \delta^t + \sum_{t=0}^{\infty} \delta^{T+2t} - \sum_{t=0}^{\infty} 2\delta^{T+2t+1} \right) \\
 &= (1-\delta) \cdot \left(\frac{1}{1-\delta} + \delta^T \cdot \left(\sum_{t=0}^{\infty} \delta^{2t} - 2 \sum_{t=0}^{\infty} \delta^{2t+1} \right) \right) \\
 &= (1-\delta) \cdot \left(\frac{1}{1-\delta} + \delta^T \cdot \left(\sum_{t=0}^{\infty} \delta^{2t} - 2 \sum_{t=0}^{\infty} \delta^{2t+1} \right) \right) \\
 &= (1-\delta) \cdot \left(\frac{1}{1-\delta} + \delta^T \cdot \left(\sum_{t=0}^{\infty} (\delta^2)^t - 2 \sum_{t=0}^{\infty} (\delta^2)^{t+\frac{1}{2}} \right) \right) \\
 &\geq (1-\delta) \cdot \left(\frac{1}{1-\delta} + \delta^T \cdot \left(\sum_{t=0}^{\infty} (\delta^2)^t - 2 \sum_{t=0}^{\infty} (\delta^2)^t \right) \right) \\
 &= (1-\delta) \cdot \left(\frac{1}{1-\delta} - \delta^T \cdot \frac{1}{1-\delta^2} \right) \\
 &= (1-\delta) \cdot \left(\frac{1+\delta}{1-\delta^2} - \delta^T \cdot \frac{1}{1-\delta^2} \right) \\
 &= (1-\delta) \cdot \left(\frac{1+\delta-\delta^T}{1-\delta^2} \right) \\
 &= \frac{1+\delta-\delta^T}{1+\delta} \\
 &= 1 - \frac{\delta^T}{1+\delta} \\
 0 &\leq \frac{\delta^T}{1+\delta} \\
 1+\delta &\leq \delta^T \\
 &\text{nicht möglich}
 \end{aligned}$$